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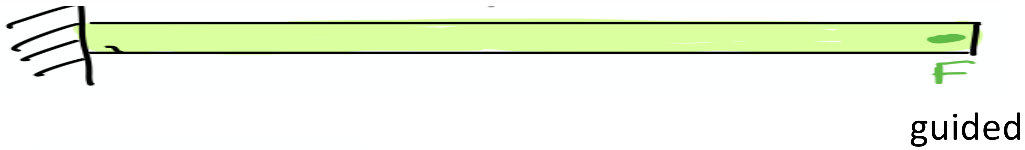
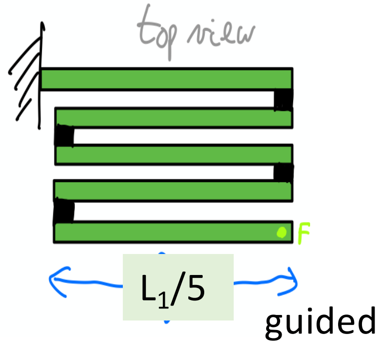
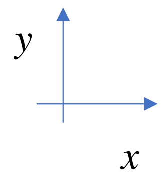






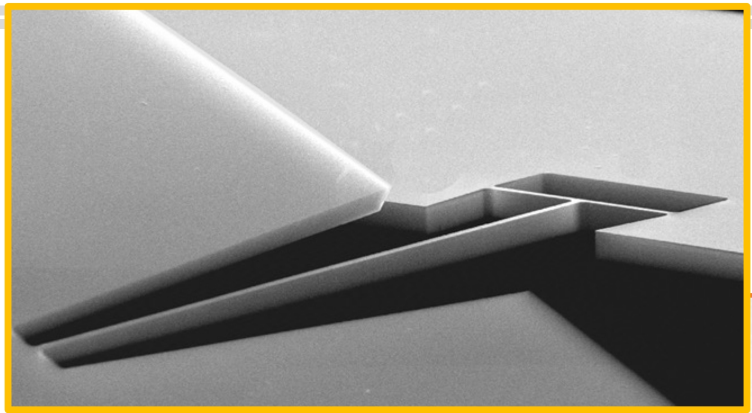
Some background on folded (serpentine) springs

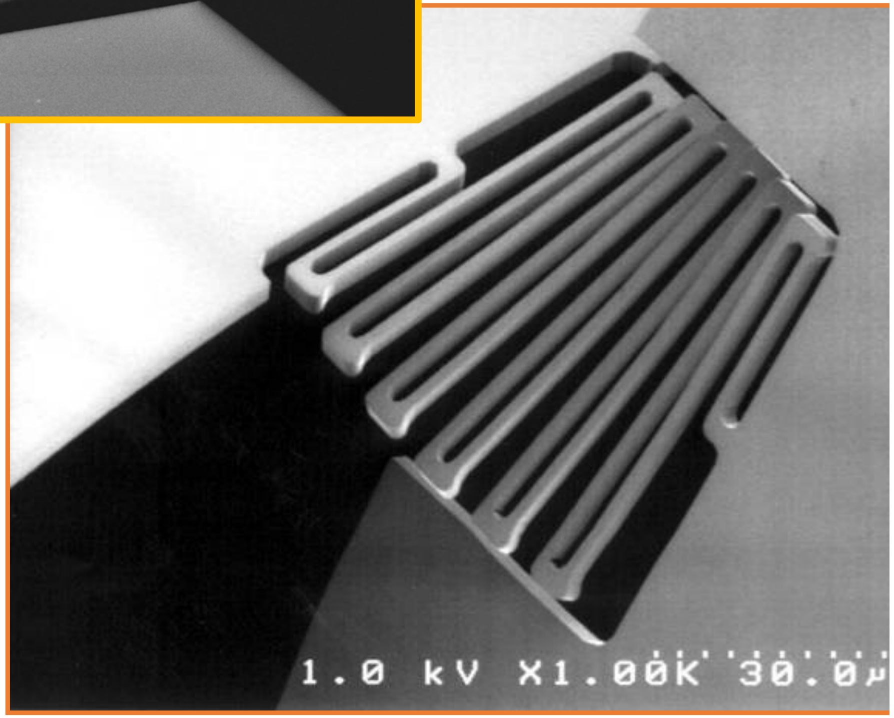
EPFL

Serpentine springs
- take less area, and
- allow for more design freedom for different modes (but adds more challenges)

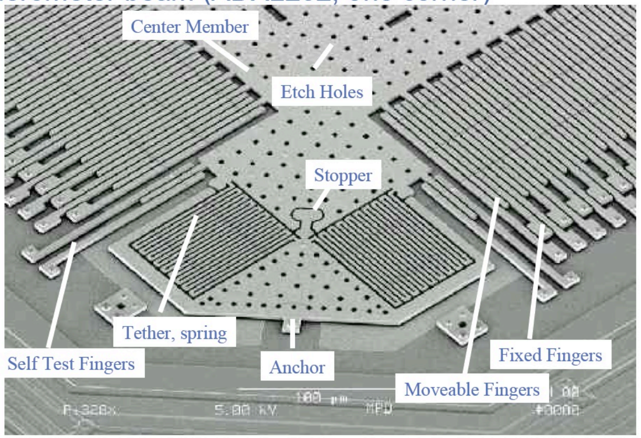
- Typically used either in overall bending or in torsion

- Need to consider bending stiffness on all axes for a proper complet design





Lucent technologies

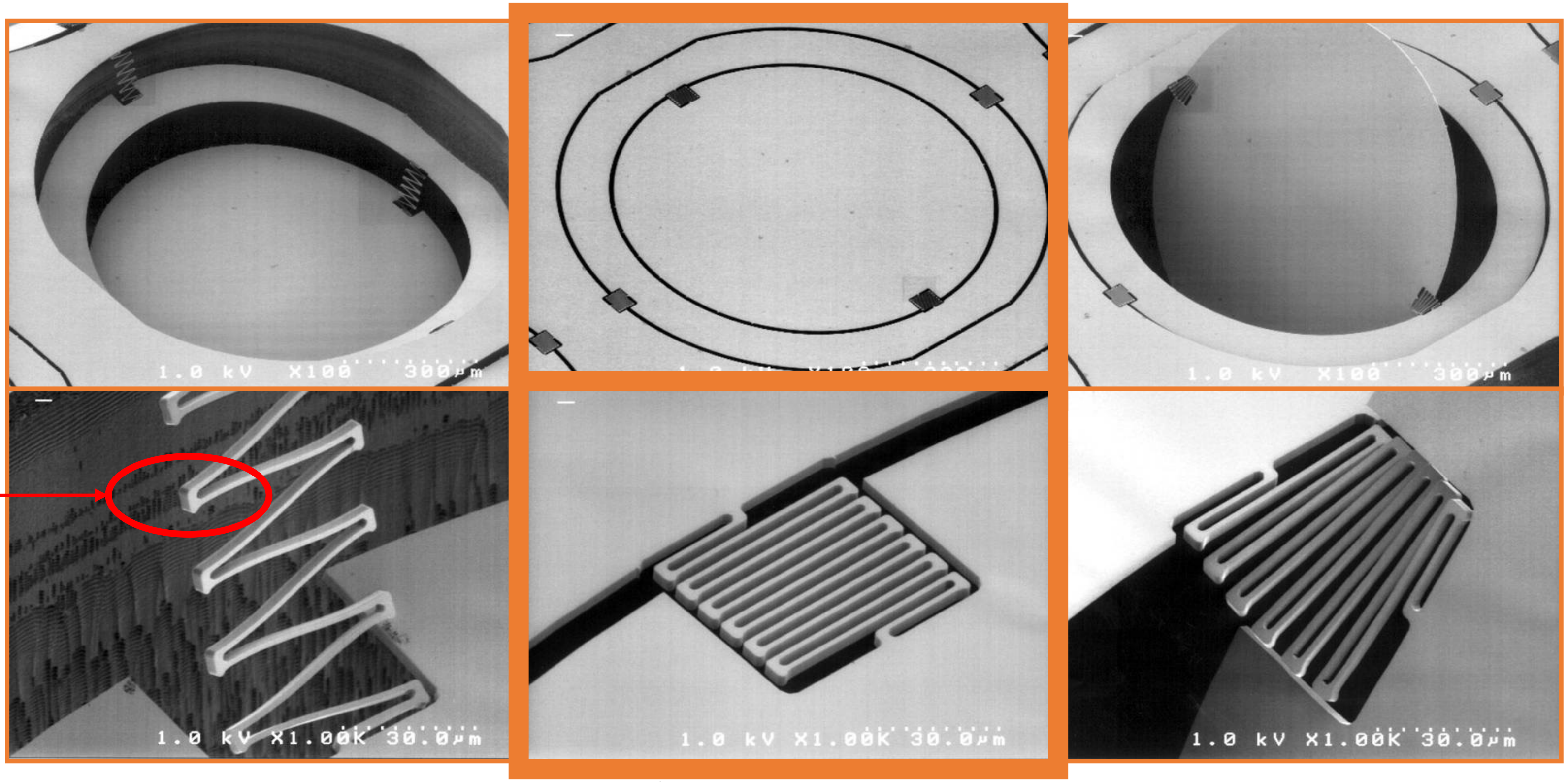
Accelerometer beam (ADXL202, one corner)


ANALOG DEVICES

Linear stretching

In torsion

Look at this shape!



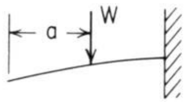
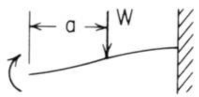
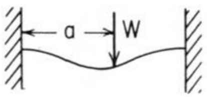
silicon springs

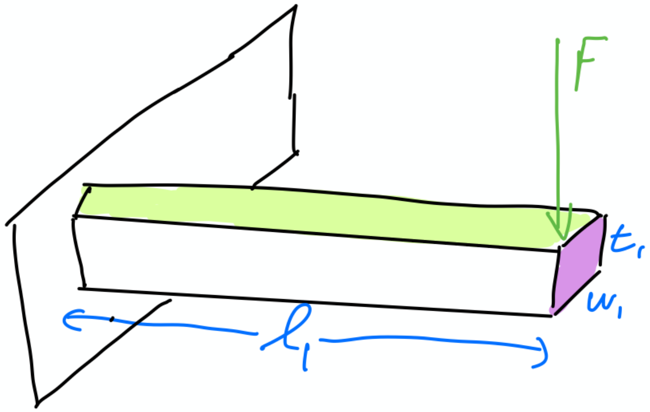
Lucent Technologies
Bell Labs Innovations

Single crystal silicon, 5 μm thick.
Segments are approximately 2 μm wide

(for background: these mirrors are for optical cross-connects).
Springs were designed primarily for torsional stiffness.

Boundary conditions matter for spring constants...

Reference no.	Boundary values	Selected maximum values of moments and deformations
1a. Left end free, right end fixed (cantilever)  	$R_A = 0 \quad M_A = 0 \quad \theta_A = \frac{W(l-a)^2}{2EI}$ $y_A = \frac{-W}{6EI}(2l^3 - 3l^2a + a^3)$ $R_B = W \quad M_B = -W(l-a)$ $\theta_B = 0 \quad y_B = 0$	Max $M = M_B$; max possible value $= -Wl$ when $a = 0$ Max $\theta = \theta_A$; max possible value $= \frac{Wl^2}{2EI}$ when $a = 0$ Max $y = y_A$; max possible value $= \frac{-Wl^3}{3EI}$ when $a = 0$ $k = \frac{3EI}{l^3}$
b. Left end guided, right end fixed  	$R_A = 0 \quad M_A = \frac{W(l-a)^2}{2l} \quad \theta_A = 0$ $y_A = \frac{-W}{12EI}(l-a)^2(l+2a)$ $R_B = W \quad M_B = \frac{-W(l^2-a^2)}{2l}$ $\theta_B = 0 \quad y_B = 0$	Max $+M = M_A$; max possible value $= \frac{Wl}{2}$ when $a = 0$ Max $-M = M_B$; max possible value $= \frac{-Wl}{2}$ when $a = 0$ Max $y = y_A$; max possible value $= \frac{-Wl^3}{12EI}$ when $a = 0$ $k = \frac{12EI}{l^3}$ 4 x stiffer!
1d. Left end fixed, right end fixed  	$R_A = \frac{W}{l^3}(l-a)^2(l+2a)$ $M_A = \frac{-Wa}{l^2}(l-a)^2$ $\theta_A = 0 \quad y_A = 0$ $R_B = \frac{Wa^2}{l^3}(3l-2a)$ $M_B = \frac{-Wa^2}{l^2}(l-a)$ $\theta_B = 0 \quad y_B = 0$ Double clamped	Max $+M = \frac{2Wa^2}{l^3}(l-a)^2$ at $x = a$; max possible value $= \frac{Wl}{8}$ when $a = \frac{l}{2}$ Max $-M = M_A$ if $a < \frac{l}{2}$; max possible value $= -0.1481Wl$ when $a = \frac{l}{3}$ Max $y = \frac{-2W(l-a)^2a^3}{3EI(l+2a)^2}$ at $x = \frac{2al}{l+2a}$ if $a > \frac{l}{2}$; max possible value $= \frac{-Wl^3}{192EI}$ when $x = a = \frac{l}{2}$ $k = \frac{192EI}{l^3}$ 64 x stiffer!



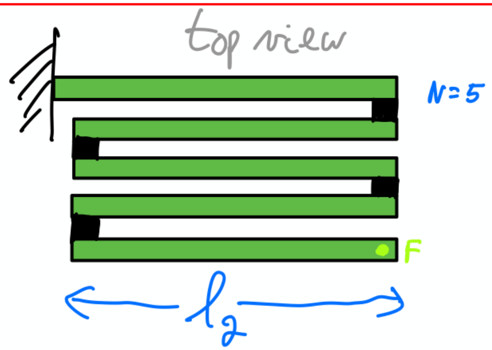
$$k = \frac{12EI}{l^3}$$

$$I_1 = \frac{1}{12} w_1 t_1^3$$

Since the mirror is attached, we will use the guided –fixed formula for a single beam

$$k = \frac{12EI}{l^3}$$

Single beam



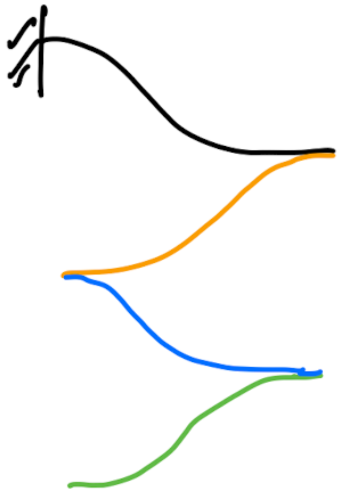
$$k_2 = ??$$

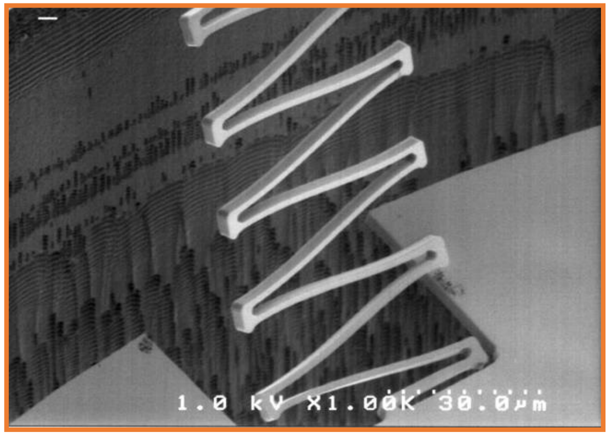
Serpentine beam, in bending

Springs in series? In parallel?

How does each spring deform?
See SEM image on slide 3

Serpentine beam. What assumptions to make about bending shape?





angle 0 at each end of the folded beams

For each segment, we also have the guided – fixed case

But now we have N beam in series, so

$$k_{serpentine} = \frac{k_{segment}}{N}$$

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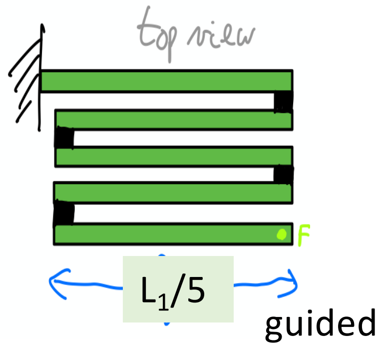

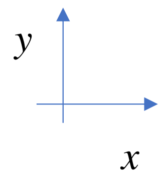

- 4. Express the side bending stiffness for force in *y* direction vs. geometry

- 5. For the 3 reasonable spring designs that gave 1N/m in z direction, what stiffness do you get in the y direction?